

An analysis of near-circular lunar mapping orbits

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Numerical investigations have been carried out to analyse the evolution of lunar circular orbits and the influence of the higher order harmonics of the lunar gravity field. The aim is to select the appropriate near-circular orbit characteristics, which extend orbit life through passive orbit maintenance. The spherical harmonic terms that make major contributions to the orbital behaviour are identified through many case studies. It is found that for low circular orbits, the 7th and the 9th zonal harmonics have predominant effect in the case of orbits for which the evolution is stable and the life is longer, and also in the case of orbits for which the evolution is unstable and a crash takes place in a short duration. By analysing the contribution of the harmonic terms to the orbit behaviour, the appropriate near-circular orbit characteristics are identified.

1. Introduction

The behaviour of satellites orbiting the Moon is mainly influenced by the non-spherical gravity field of the Moon. Many lunar missions demand a long operational life to conduct scientific experiments. For example, the Indian Moon mission Chandrayaan-1 is conceived to have an operational life of two years in a circular polar orbit. To achieve this operational life, regular orbit maintenance to keep the spacecraft in the envisaged orbit is required. These manoeuvres require a large amount of fuel. The limitations on the availability of fuel in the spacecraft for this purpose, restricts the operational life of the moon orbiting satellites. It is desirable to use the lunar gravity field to design orbits that require minimum fuel for maintenance. Study (Cook 1991) indicates that by appropriate choice of orbit characteristics the natural variations in the orbits can be reduced. Frozen orbits, for which there is no long-term change in eccentricity and argument of periapsis, were suggested in this study. To obtain a frozen orbit one chooses an eccentricity and an argument of periapsis for a given semi-major axis and inclination.

Cook (1991) developed closed form expressions to find the eccentricity for a given inclination and

this combination provides stability in the orbit evolution. These closed form solutions are the functions of the lunar gravity field which consider only zonal terms. He identified three inclinations, which provide frozen orbits for 150 km circular orbits. Park and Junkins (1995) used the same closed form expressions and arrived at frozen orbit parameters with updated gravity models. They identified an inclination of 101.5° as the candidate for 100 km circular orbits. The limitations of the frozen orbit concept for circular orbits are discussed in literature (Cook and Sweetser 1992) and strategies for orbit maintenance are proposed. With tighter mission specifications on eccentricity and argument of periapsis frequent manoeuvres are envisaged. Also, frozen orbit parameters are obtained using the same formulation with the zonal terms of the LUN60D gravity model and it is concluded therein that the determination of a polar circular frozen orbit is inconclusive (Konopoliv *et al* 1993). A severe variation in the life times predicted with different gravity fields, is also reported in the literature.

Accurate gravity model is required especially with low altitude orbits. Gravity models for the Earth with zonal terms only, yield reasonably good approximations to the actual field in orbit

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Table 1. Gravity models and orbital life time estimates.

Model	Life time for 100 km circular polar orbit (days)
LUN60D (60 × 60)	193
GLGM2 (70 × 70)	> 2 years
LP100J (100 × 100)	160

Table 2. Harmonic coefficients and the orbital life time.

LP100J degree × order	Life time for 100 km circular polar orbit (days)
100 × 100	160.400
70 × 70	161.569
50 × 50	160.611
40 × 40	155.785
30 × 30	130.646
20 × 20	222.410

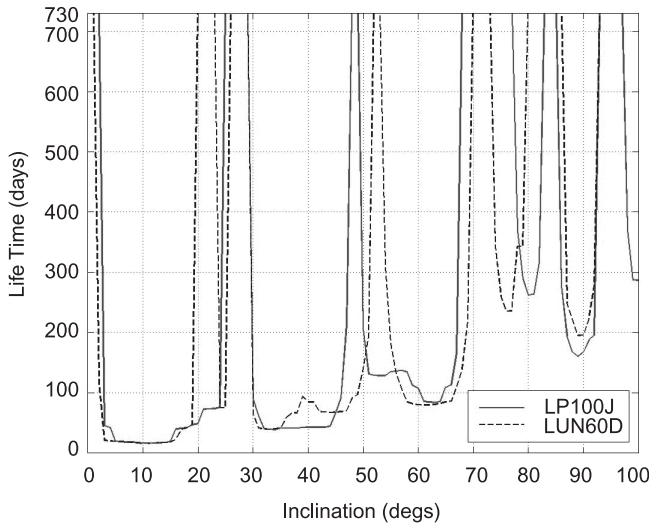


Figure 1. Impact of inclination on the orbital life time.

evolution studies. However, with the moon's gravity field heavily dependent on longitudinal variations, it becomes necessary to include non-zonal terms also in the orbital evolution process. In this paper, we identify a gravitationally stable lunar mapping orbit which is circular and near-polar by including the full gravity model. Because of the limitations of the closed form solutions for circular orbits and the necessity of non-zonal terms for accurate orbit evolution, investigations are carried out using Cowell's integration procedures. The life times of the spacecraft are assessed for various orbit characteristics, and a stable orbit characteristic is identified for circular near-polar orbit. Further, the spherical harmonic terms (zonal as well as tesseral) that make major contributions to the orbital evolutions, are identified.

2. Simulation model

The simulations are carried out using the equations of motion representing the inverse square field together with the non-spherical lunar gravity field. The lunar gravity potential, U , is modeled in spherical harmonics using the expression,

$$U = \frac{GM}{r} \sum_{l=0}^{\infty} \left(\frac{a_e}{r} \right)^l \times \sum_{m=0}^l \{ \bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda \} \bar{P}_{lm}(\sin \phi),$$

where r is the selenocentric radius, ϕ and λ are the selenocentric latitude and longitude respectively; \bar{C}_{lm} and \bar{S}_{lm} are the normalized potential coefficients; \bar{P}_{lm} are the normalized associated Legendre functions of degree l and order m ; a_e is the lunar equatorial radius; G is the universal constant of gravitation, and M is the lunar mass. Gravity models provide the potential coefficients. Other perturbing forces from the Earth and the Sun are not included, because they do not cause much variation in life time assessment studies.

3. Lunar gravity models

The gravity field for the Moon has been determined by the measurements carried out by several missions. Though many gravity field models do exist in the literature, three gravity models represent the ongoing evolutionary process. The first elaborate and complete model LUN60D (Konopoliv *et al* 1993), a 60 × 60 gravity model, was determined by earlier lunar orbiter and Apollo missions. However, many of these missions were with low inclination lunar parking orbits and the measurements covered only the equatorial region of the near side (visible to the Earth). With more data from the highly inclined orbit, Clementine mission provided GLGM2 (Lemoine *et al* 1997), a 70 × 70 gravity model. Lunar Prospector Mission was in polar orbit and covered the earlier uncovered regions of near side, and based on these data gravity models of degree and order of up to 160 have been developed (Konopoliv *et al* 1998). LP100J is one such 100 × 100 model.

In an exercise to determine the choice of gravity models for this study, life times of the moon orbiting satellites are assessed with the three gravity models. Table 1 presents the orbital life times of a 100 km circular polar orbit with these gravity models. The huge differences indicate the inadequacy of the models. Of these a 100 × 100 model LP100J and LUN60D are considered for a study in which the predicted life times are close to the

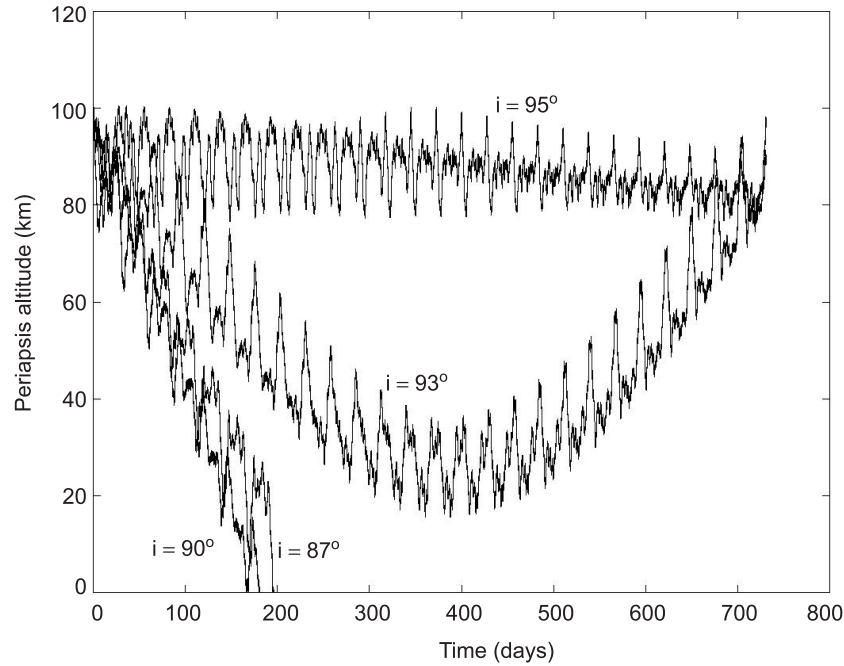


Figure 2. Evolution of perapsis altitude for different inclinations.

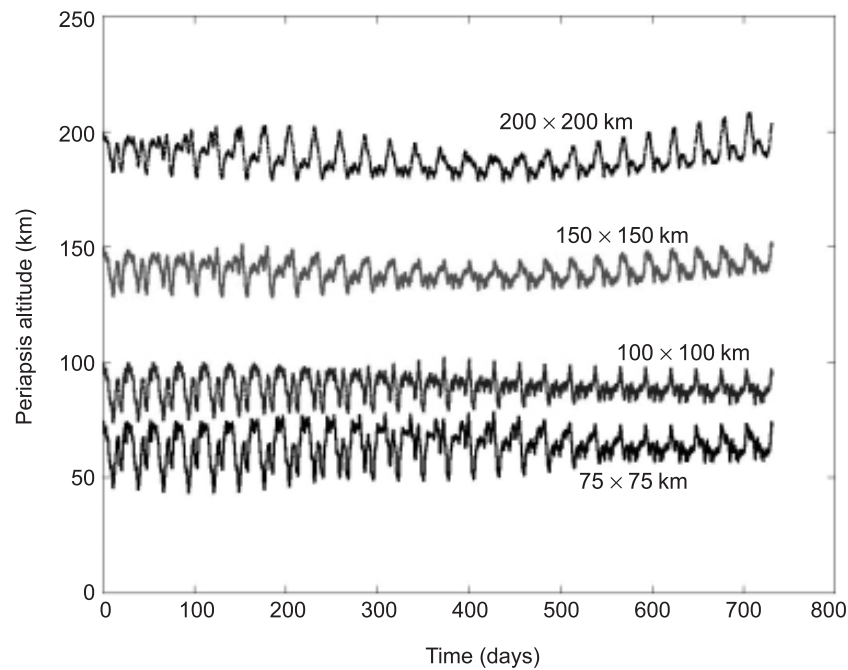


Figure 3. Evolution of perapsis altitude for different orbit sizes.

ones reported in the literature (Lozier *et al* 1998). The investigations being numerical, it is natural to attempt to reduce the gravity model coefficients without compromising the accuracies to save computational time. LP100J, a 100×100 model, is considered for this exercise. Table 2 predicts the orbital life time for a 100 km circular polar orbit, for various models that are truncated LP100J models. Other initial orbital characteristics (argument

of perapsis, right ascension of ascending node and mean anomaly) are set to zero. A 50×50 model predicts the life time with less than 0.5% error, and hence considered for further investigations. Prediction accuracy suffers severely with any further reduction of the model. Though, the analysis for choice of truncated model is provided for one orbit, similar behaviour is expected for other orbits.

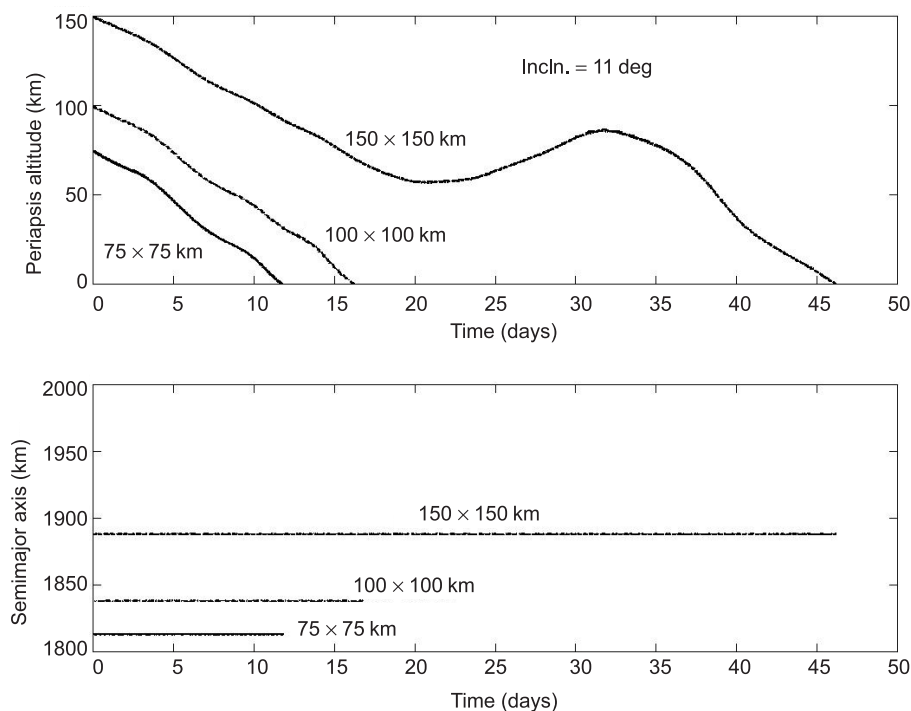


Figure 4. Evolution of periapsis altitude and semi-major axis.

Table 3. Impact of right ascension of ascending node on orbit evolution.

RAAN ($^{\circ}$)	Inclination = 90° Life time (days)	Inclination = 95°	
		Life time (days)	Minimum periapsis altitude (km)
0.0	167.45	> 2 years	74.0
30.0	167.42	> 2 years	52.78
60.0	172.01	> 2 years	57.42
90.0	174.94	> 2 years	50.31
120.0	203.25	> 2 years	57.30
150.0	179.32	> 2 years	60.69
180.0	175.92	> 2 years	54.18
210.0	181.35	> 2 years	53.20
240.0	183.65	> 2 years	56.80
270.0	180.45	> 2 years	57.20
300.0	180.74	> 2 years	69.42
330.0	165.64	> 2 years	70.53

4. Behaviour of near-circular lunar mapping orbits

The spacecraft in a 100 km circular polar orbit has an orbital life time of about 160 days only due to the gravity field. Orbit maintenance manoeuvres are required to keep periapsis within the mission specifications (± 20 km in the case of Lunar Prospector (Lozier *et al* 1998)). As pointed out earlier, appropriate initial orbit characteristics can extend the life and reduce the maintenance cost. With the orbit size and shape fixed as circular, the parameters which can be changed are inclination

and the right ascension of ascending node. The contribution of these parameters to the orbital behaviour in the presence of the gravity field is studied. Appropriate values for these parameters are selected. The contribution of spherical harmonics of the lunar gravity field to the orbital evolution is analysed.

4.1 Impact of inclination

The orbital life times of a 100 km circular orbit has been assessed for different inclinations, and are plotted in figure 1. The right ascension of

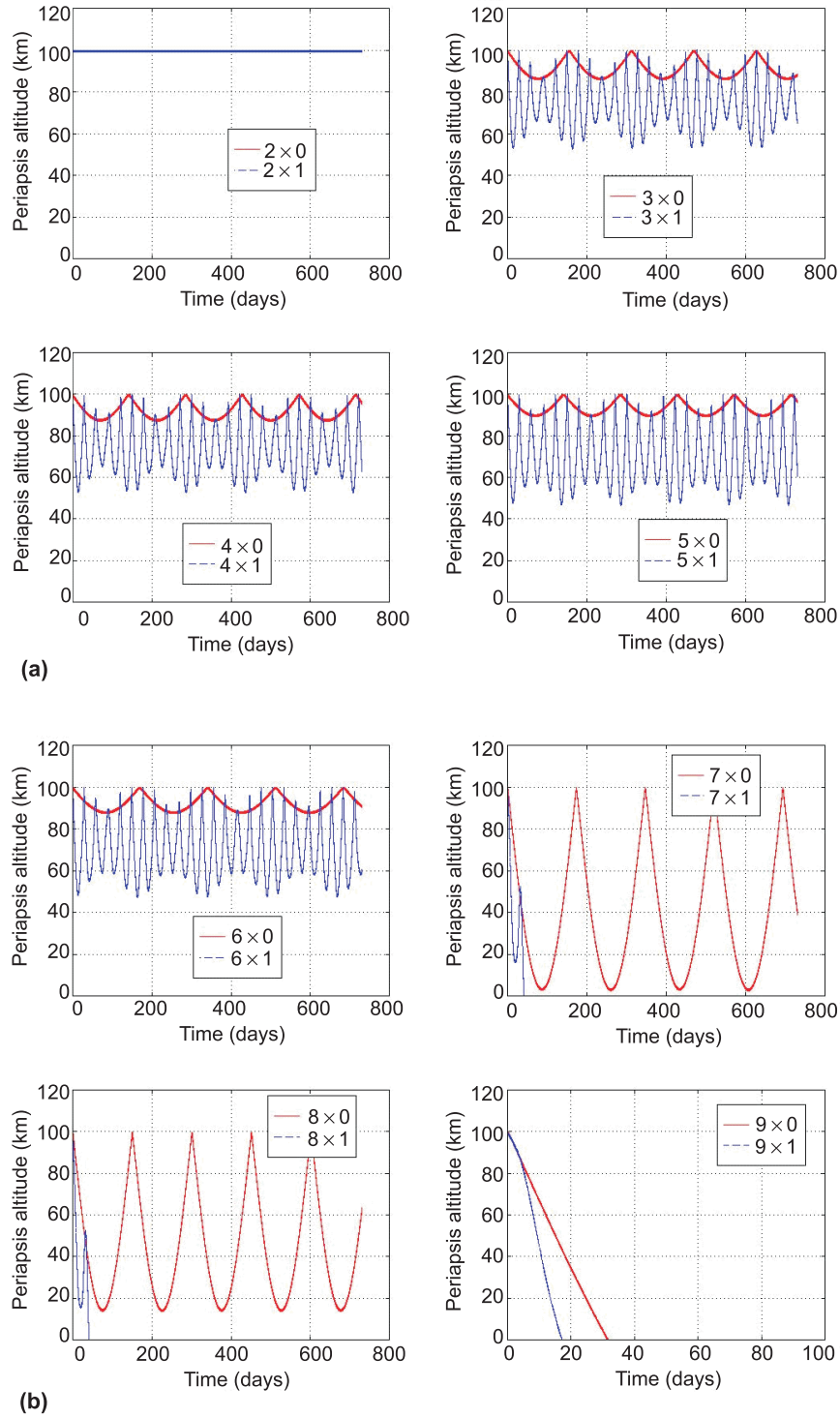


Figure 5. Contribution of spherical harmonics to the orbit evolution ($i = 11^\circ$).

ascending node is set to zero for this figure. The strong dependence of the orbital life time on orbital inclination is clearly seen. The life time estimates vary from a minimum of 16 days to more than 2 years duration for the same size of the orbit. The trends are the same for both the gravity models under consideration. It is interesting to note that there exist a few regions of inclinations that

yield long life times. The minimum orbital life time occurs when the inclination is about 11° . The orbital periapsis altitude behaviour for some inclinations around 90° is plotted in figure 2. The stability of the orbit for $i = 95^\circ$ can be noticed while for $i = 90^\circ$, the orbit decays rapidly. The maximum absolute variation is about 26 km only in two years for $i = 95^\circ$. This orbit inclination keeps the

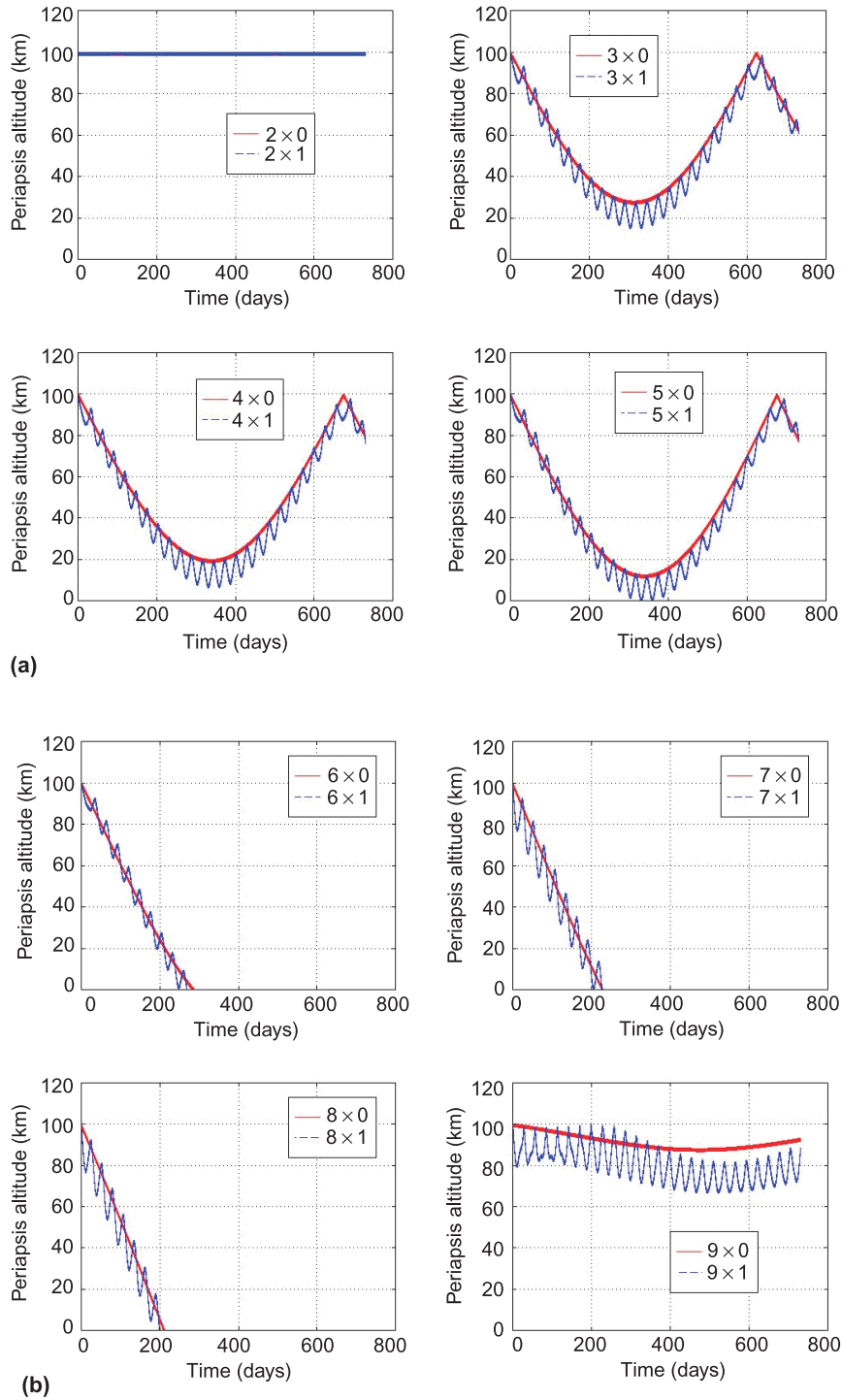


Figure 6. Contribution of spherical harmonics to the orbit evolution ($i = 95^\circ$).

variation of periapsis altitude at a very low level for other circular orbits of 75, 150 and 200 km also. This is illustrated in figure 3.

The evolution of the periapsis altitude and the semi-major axis for $i = 11^\circ$, the minimum life time case, is given in figure 4. Though the semi-major axis remains constant in all the cases, the severe eccentricity variation due to the lunar gravity accelerates the impact of the spacecraft. For high

as well as for low circular orbits, a similar trend is noticed.

4.2 Impact of right ascension of ascending node (RAAN)

Proper choice of this parameter could further extend the life and reduce the maintenance cost. This parameter is measured in the IAU (International

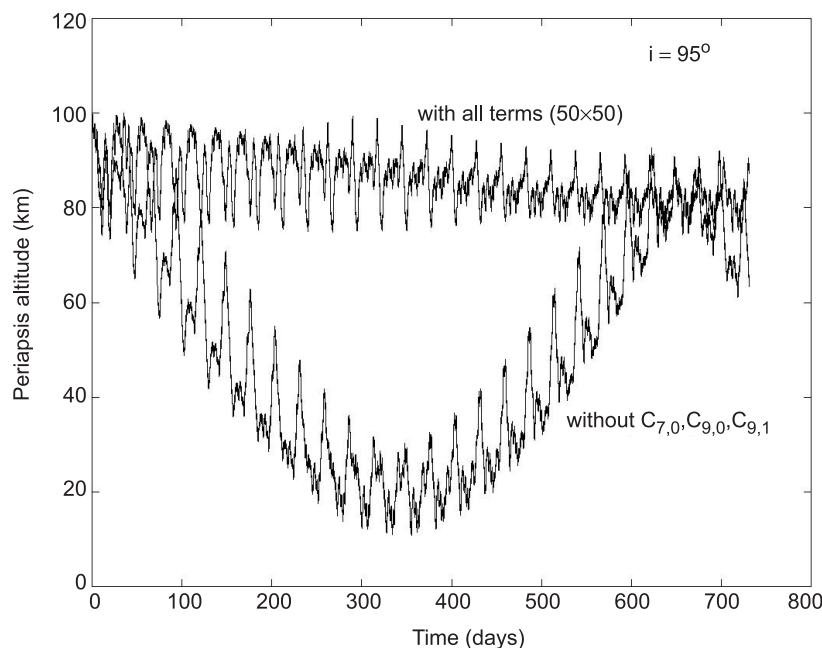


Figure 7. Impact of the spherical harmonics $C_{7,0}$, $C_{9,0}$, $C_{9,1}$ on the periapsis altitude evolution.

Astronomical Union) coordinate frame. For inclination of 95° , when RAAN is in the range of $(300$ to $360)^\circ$, the minimum periapsis altitude in two years is about 70 km (see table 3). This kind of periapsis evolution reduces not only the cost of maintenance but also the frequency of the maintenance, and hence this range is preferable. For the polar orbits, a value of about 120° for RAAN extends the life by about 35 days. Force model in these investigations include the third body effects of the Earth and the Sun. It is pointed out that RAAN does not affect the orbit evolution in the absence of non-zonal terms.

4.3 Contribution of harmonics

Numerical investigations are carried out to find out the contribution of the harmonics. The inclination $i = 11^\circ$ for which rapid decay takes place and the inclination $i = 95^\circ$ for which the long life of the craft exist, are considered for the study. These are presented in the figures 5 and 6. Clearly, the spherical harmonic terms $C_{7,0}$, $C_{9,0}$ and $C_{9,1}$ make major contributions to the orbital evolution of all the cases. When $C_{7,0}$ is included in the force model the periapsis altitude variation is more than 90 km. Inclusion of $C_{9,0}$ makes it to crash and the inclusion of $C_{9,1}$ speeds up the crash. Also, the tesseral harmonics cause medium periodic (with periods that are less than the moon's orbital period) variations in the periapsis altitude. These variations must be estimated as accurately as possible to devise a maintenance strategy that involves estimation of the magnitude of velocity

requirements and frequency of the manoeuvres. Similar trends are observed for other inclinations also. For high life time cases ($i = 95^\circ$), the absence of these harmonics results in low periapsis altitudes during the orbital evolution. This is illustrated in figure 7. These terms accelerate the periapsis altitude downwards in low life time cases and decrease the rate of change in the periapsis altitude in high life time cases.

5. Conclusions

The impact of inclination on the evolution of periapsis altitude is assessed. An inclination of 95° has the least variation in the periapsis altitude and this variation lies within ± 26 km. This choice reduces the maintenance cost considerably. The orbital life time is minimum (16 days) for an inclination of 11° for low altitude orbits. The specific spherical harmonics that cause this phenomena are $C_{7,0}$, $C_{9,0}$ and $C_{9,1}$. These terms help to stabilize the periapsis altitude for the 95° case while being the cause for early crash in the cases of low orbital life times. A proper choice of right ascension of ascending node (0° for 95° inclination case) was also found to improve the orbital lifetime and reduce the maintenance cost.

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